Making the FCC's First Combinatorial Auction Work Well

Comment on DA 00-1075,
"COMMENT SOUGHT
ON MODIFYING THE SIMULTANEOUS MULTIPLE ROUND AUCTION DESIGN TO ALLOW COMBINATORIAL (PACKAGE) BIDDING"
Report No. AUC-00-31-G (Auction No. 31)

by

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Table of Contents

1. Overall Comment

It is appropriate for the Commission to use combinatorial bidding in auctioning multiple related licenses. Thus, this proposal is a major (and overdue) step in the right direction. However, many details of this proposal need attention, correction, or enhancement. We also note that the detailed rules are interrelated and cannot be considered independently. Below, we discuss a number of these details and try to identify some unifying principles for combinatorial auctions. The overall goal is to design an inconsistency-free, efficient auction that will eliminate the exposure problem of bidders with synergies for combinations of licenses while minimizing the threshold problems of bidders without such synergies. Furthermore, since this is the FCC's first combinatorial auction and since future auctions are likely to involve more complicated situations, this auction should be designed in a way that generalizes gracefully. The rule clarifications and changes that we recommend are not dramatic, but they will enable a more efficient auction and help avoid potential problems.

2. Qualifications

The authors of this comment are independent scholars who have studied auctions, in general, and combinatorial auctions, in particular. We do not represent any bidder or the FCC. We are co-authors of the seminal paper on the computational issues that arise in the design of combinatorial auctions like this one.[1] That paper specifically demonstrates that computational problems will not arise in the proposed auction. In addition, Professor Rothkopf has published over 20 scholarly papers on auctions including papers that deal with several specific issues related to this auction design[2] and Professor Pekec is an expert in the design and analysis of combinatorial optimization models.

3. Make the Auction Rules Easy to Extend to Future Auctions

Because it is easier to make small changes in auction format than larger ones, it is desirable for the commission to design the particulars of the rules in this auction in a way that will generalize gracefully to more complicated situations to follow. In this auction, it is apparently clear to the Commission which combinations are of economic interest, and it is right and appropriate that the Commission is allowing bids on the important combinations. The row-column format as well as the small number of licenses

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[1] Michael H. Rothkopf, Aleksandar Pekec and Ronald M. Harstad, "Computationally Manageable Combinational Auctions," *Management Science* **44**, pp. 1131-1147, 1998.

[2] Michael H. Rothkopf and Ronald Harstad, "Modeling Competitive Bidding: A Critical Essay," *Management Science* **40**, pp. 364-384, 1994 deals with realistic analysis of auctions.

Michael H. Rothkopf and Ronald M. Harstad, "On the Role of Discrete Bid Levels in Oral Auctions," *European Journal of Operational Research* **74**, pp. 572-581, 1994 deals with analyzing the effects of varying bid increments.

Ronald M. Harstad and Michael H. Rothkopf, "Withdrawable Bids as Winner's Curse Insurance," *Operations Research*, **43**, pp. 983-994, 1995 provides the rational for the bid withdrawal penalties used in the FCC auctions.

Benjamin F. Hobbs, Michael H. Rothkopf, Laurel C. Hyde, and Richard P. O'Neill, "Evaluation of a Truth Revealing Auction for Energy Markets with Nonconcave Benefits," To appear in *Journal of Regulatory Economics* evaluates a possible auction form for auctions with synergies between items being offered.

guarantees that the Commission will have no computational trouble in determining the provisional winning bids. In future auctions, however, it may be less clear what the desirable combinations are. We note that Rothkopf, Pekec and Harstad,[3] contains an analysis of kinds of combinations for which winner determination is guaranteed not to cause computational difficulties.[4] We suggest that the Commission plan in future auctions to let the bidders prioritize combinations of economic importance to them. The work presented at the Wye River Conference by Rothkopf and Park suggests that this can be made to work.[5] Since the Commission seems intent upon retaining its multi-round auction format, the rules for combinatorial auctions should be developed in such a way that they generalize gracefully to auctions in which bidders can select arbitrary combinations.

4. A Proposal for Determining Minimum Eligibility-Conferring Bids

Bidders should get activity credit for having standing high bids or for making bids that advance the auction. The choice of minimum bid that will confer eligibility is important since bidding, even on a license or combination that is not a provisional winner, may confer eligibility. In its request for comments, the Commission offers several alternatives for minimum acceptable bids. None of the alternatives offered seems to be ideal. Of the alternatives specifically discussed, the one we like best is the final one. It proposes allocating the total amount needed to beat the provisional winners. However, a problem with this proposal is that the allocation mechanism can be improved. The allocation should be proportional to the amount the bid would have to be increased to make it a provisional winner and not to its MhzPops. To be fair, the allocation should be distributed over all provisionally non-winning bids that would become winners, figured in the way that minimizes the size of the needed increase. (The makers of provisionally winning bids have no reason to contribute.) The reason for doing the allocation this way is that bidding activity determines eligibility, which should depend upon whether a bidder's bids are moving the auction along.[6] The minimum eligibility-conferring bid increase should be the smallest possible increase that moves the bids towards becoming provisionally winning on the assumption that other non-provisionally winning bids do their share (or the bid increment X specified by the Commission[7], whichever is larger[8]). If no one is willing to make a proportional increase on any of the non-provisionally winning bids involved, there is no hope that the bids on the licenses or combinations in question can become provisional winners. Thus, we believe that these are the smallest bid increments for which bids should count towards meeting activity requirement for

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[3] *Op. Cit.*

[4] For example, allowing bids on packages containing at least one license from each region would not complicate the auction. There are 729 possible packages of this type and, under current proposal, the FCC allows bids on only three such packages (global package bid and two national package bids). Another possibility is to (also) allow bids on packages consisting of any collection of regional package bids. However, if the number of allowed package bids becomes large, we would recommend limiting number of package bids submitted by bidder. If the Commission decides to allow bids on additional packages, we would like an opportunity to suggest means for doing so.

[5] See http://combin.fcc.gov/fcc_wye.pdf.

[6] As discussed later in this section, there is reason to allow smaller bid increments as well, provided that no eligibility is conferred by these smaller bids and that the bidders know this in advance.

[7] e.g. 5% of the current bid

[8] For the provisional winning bids, this amounts to the X that the Commission specifies.

conferring eligibility. An appendix develops a formal mathematical statement of the formula that we suggest the Commission use. Here is an example that illustrates the use of the calculation we propose.

> Suppose that the following are the high bids at the end of a round and that the commission specified X is 5% of the current bid:
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> Global Package - $480
> National 10 MHz Package - $200, National 20 MHz Package - $250
> Regional package bids (in numerical order): $40, $60, $100, $110, $80, $80
> Individual bids by region (in numerical order), 10 MHz:
> $10, $20, $30, $40, $25, $25
> Individual bids by region (in numerical order), 20 MHz:
> $20, $40, $60, $80, $50, $50
>
> In this situation, the 10 MHz national package and the individual licenses for the 20 MHz licenses are the provisional winners with a total revenue of $500. Thus, the minimum bid increase on these licenses/packages is 5% of the current bid.
>
> There is no retained global bid. To earn activity credit, a bid on the global license would have be at least $504 ($24 is 5% increase here and this is larger than $500-$480=$20).
>
> To earn activity credit, the bid on the 20 MHz national package would have to increase by $300-$250=$50. Thus, the bidder on this package will be faced with a 20% minimum bid increase.
>
> As a final but more complicated example, consider the situation faced by the $20 bid for the 10 MHz license in Region 2. The highest sum of the nonconflicting bids that includes the $20 bid on the 10 MHz license in region 2, is $480. This maximum is obtained by considering regional bids in regions 1, 3, 5, and 6 and the individual bids in regions 2 and 4. Thus, if the sum of these bids were to increase by more than $500-$480=$20, the bid on 10MHz license in Region 2 would become provisionally winning. Note that the bids on the 20 MHz individual licenses in regions 2 and 4 are already provisionally winning. The other non-provisionally winning bids total $360. Thus, to do its proportional share, the bid on the 10 MHz license in Region 2 would have to bear $20/$360 (=5.56%) of the needed $20 increase, i.e., would have to increase by $1.11.

No bid that is smaller than the minimum calculated this way should count as activity for the purposes of conferring eligibility. To do so could convert obviously noncompetitive retained bids into eligibility parking lots. This would slow the auction and distort the economic message being sent by the bids. However, since the minimum bid increments calculated in this way can be high if a retained bid is far from becoming a provisional winner, the Commission should allow smaller bid increments to be used. This would allow a bidder on an individual license or package who can contribute less than a proportional share to overcoming the lead of a larger package bid to do so.

5. Activity Credit, Eligibility and Withdrawal Rules

5.1 Grant Activity Credit Sparingly

On one hand, we would like only seriously competitive bids as defined in the previous section and retained bids to confer eligibility. On the other hand, we believe that no bidder should lose eligibility without knowing that it is to be lost. A bidder whose

eligibility is conferred by a retained bid that is not provisionally winning should not lose the right to bid again on it if her retained bid is topped. She should retain activity credit for that retained bid in future rounds for as long as she bids on it in each round. For example, assume that the Commission is asking for 5% bid increments and that bidder A has a retained bid of $100 on a license that is not provisionally winning. Suppose that a bid of $200 would be required to confer eligibility and that some other bidder bids $105 in the next round.[9] In the following round, bidder A should be able to bid $110.25. If she chooses to do so, she will gain activity credit for the bid whether or not it becomes a retained bid. For example, if bidder A makes this bid and is topped by a bid of, say, $115.50, she would be eligible to make a bid of $121.28 in the next round, and so on.

Thus, our proposal is to grant initial activity credit for bidding on a particular license/package only if the bid on it is eligibility-conferring or becomes a retained bid. Once a bid becomes a retained bid, however, we propose that the bidder continue getting activity credit on the licenses/package as long as she bids on it in every round once it has been topped.[10]

This approach is consistent with click box bidding. The bidder should be given clearly labeled boxes for eligibility-conferring and non-eligibility-conferring bids. Where future eligibility depends upon continued bidding on a license, this should be clearly indicated.

In summary, we propose a small change in the activity granting mechanism that will minimize occurrences of eligibility parking lots and, at the same time, ensure that bidders are never surprised by the loss of eligibility. According to our proposal:

- Any retained bid confers eligibility.
- Any bid calculated as described in the previous section as large enough to be eligibility-conferring does so too.
- To retain the no-surprises aspect of eligibility loss, if a retained bid is topped, its maker can retain the eligibility it conferred by continuing to bid actively on that license or package.

5.2 Use only Non-Conflicting Bids to Calculate Required Eligibility

In addition, we favor the use of the maximum possible provisionally winning number of bidding units to measure eligibility requirements. Thus, only one of a bidder's set of self-conflicting bids in a round would count against its eligibility allowance. We note that this greatly limits the need for bidders to use OR bids.[11] We also note that this could result in a bidder having more standing bids (but not more provisionally winning

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[9] The $105 bid, like any other valid bid, would count towards its maker's maximum eligibility for the round. However, the bidder will get activity credit for making this bid (which is lower than the minimum eligibility-conferring bid of $200) only if she has been bidding on it continuously since having held the retained bid on the same license in a previous round. Of course, if it becomes a retained bid, the bidder will get activity credit for it.

[10] Note again that the minimum bid on any provisionally winning licenses or package is eligibility-conferring. The rule discussed here is an exception only for retained bids that are not provisionally winning.

[11] Any two conflicting bids are effectively OR bids under this rule. For example, a bidder could submit a bid on a national package and one on a regional one in complete confidence that both cannot be accepted. However, this would not cause any of the problems associated with allowing general OR bids discussed below.

bids) than her eligibility. For example, a bidder with only enough eligibility for a regional license could have retained bids on both a regional package and a single license in that region (but not on one in another region). One or neither of these two retained bids, but not both, could be provisional winners. We see no problem with this, but the rules should make clear that this does not increase a bidder's eligibility. Thus, the bidder in our example would not have increased her eligibility by holding two standing bids.

5.3 Allow Withdrawals of Non-Provisionally Winning Bids, But Only to Enable Other Bidding

The use of combinatorial bidding greatly reduces the legitimate need for bid withdrawals during an auction. In particular, withdrawals of provisionally winning bids are no longer needed to escape failed aggregation attempts. Therefore, provisionally winning bids should not be withdrawn without severe penalty. However, bids that are retained but are not provisionally winning may be a burden to a bidder who wants to bid for something else. Therefore, withdrawal without penalty of such retained, non-provisionally winning bids should be allowed but only by a bidder making a new eligibility-conferring bid that gives the same or greater eligibility. (Allowing the withdrawal of retained bids for other reasons serves no important purpose but could prolong the auction and certainly invites gaming and signaling.) In order to avoid misleading other bidders who are interested in the package upon which the withdrawal is planned, the bidder wishing to make such a withdrawal should be required to announce it in advance.

For example, a bidder with a retained but not provisionally winning bid on the 20 MHz license in region 1 may, before the beginning of a round, announce the withdrawal of that bid. If she does so, she must make an eligibility-conferring bid in the following round on some other 20 MHz license or on some national, regional or global package.

If a bidder announces a withdrawal of a retained but not provisionally winning bid, the previous high bidder on the license or combination should be allowed to reinstate her prior bid. If she does not choose to do so, the bid on the license would revert to the FCC at its original minimum bid. Her decision would be announced before the next round of bidding.

6. Enhancement and Clarification of the Default Rule

Default in a combinatorial auction has more far reaching consequences than does default in an auction of single items. In particular, a default in a combinatorial auction could affect the award of many other licenses and be used strategically to do so. This should be discouraged. The Commission should increase its post-auction default penalty substantially beyond 3% plus the shortfall in a resale. In addition, it should increase its deposit requirements to insure that default payments are, in fact, collected. A possible rule change would be for the Commission to raise the 3% post-auction default premium from 3% to 25% and to require that bidders with provisionally-winning bids raise their deposits to 25% of their bids. In conjunction with this, the Commission might consider paying interest at the Treasury bill rate on deposits, or perhaps, just on the increased amount of deposits.

The proposed default rule is ambiguous with respect to the treatment of multiple defaults that are resold jointly. It is critical that this ambiguity be resolved. It is desirable

that in the event of multiple defaults that the FCC be able to resell the defaulted licenses jointly so that bidders in the resale can realize synergies without exposure risks. Therefore, we propose the following clarification of the default rule: If two (or more) packages default and are resold jointly, and if in this resale they are won by one bid that covers both (or all), then for the purpose of calculating default payments, resale amounts will be allocated in proportion to the original sale amounts.

Two examples (in which we use the 3% penalty that we are recommending be increased) will help make this clear. First, suppose that a 20 MHz national package and a 10 MHz national package are sold for $600 million and $200 million, respectively, and both default. Suppose further that in a resale the global package wins for $500 million. Then the 20 MHz defaulter would owe 3/4 [=600/(600 + 200)] of the $300 million shortfall plus 3% of $600 million less 3/4 of $300 million for a total of $235.25 million. The 10 MHz defaulter would owe 1/4 of the $300 million shortfall plus 3% of the increased post-auction default charge of $200 million less 1/4 of $300 million for a total of $78.75 million.

The rule of proportionality should also be applied in more complicated situations. For example, suppose bidder A bids $100 and then defaults on the combination of licenses 1 and 2, and bidder B bids $200 and then defaults on the combination of licenses 3 and 4. Suppose further that in a resale, the Commission allows bids on any combination of licenses and that one bidder wins the combination of licenses 1 and 3 for $50 and another wins the combination of 2 and 4 for $100. Then, bidder A would be responsible for 1/3 of the $150 total shortfall and would owe $51.50, and bidder B would be responsible for 2/3 of it and owe $103.

One other clarification of the default rule might possibly help the Commission's legal position in default bankruptcy situations. The Commission should make clear that high bids confer an obligation to the amount of the bid and, in the event of default, to the default penalty as well. The amount the Commission receives from the resale should be explicitly labeled as mitigation of damages and not a reduction in the penalty itself.

7. The Auction Needs a Foolproof Rule for Breaking Ties

Click box bidding makes tie bids or combinations of bids more than minimally likely. For this auction as well as for future more general combinatorial auctions, the Commission will need a simple, trouble free rule for breaking ties on bids involving combinations. The rule is needed to guarantee fairness and avoid litigation in any possible situation involving ties. We propose the following rule, which, on the assumption that a serial computer time stamps bids, can be proven to be unambiguous, fair, and trouble free. Ties are broken, as before in favor of the earliest bid. For the purposes of this rule, the bid on a collection of non-conflicting bids is assigned the time stamp of the LAST bid that composes it. (In some cases, the time stamp involved may carry over from previous auction rounds.) An example using the following table illustrates this:

Bid	Licenses Covered	Amount	Time Made
1	A and B	$100	1:00 pm
2	C	$50	3:00 pm
3	A	$60	2:30 pm
4	B and C	$90	2:00 pm

Bids 1 and 2 are tied with bids 3 and 4. Both pairs offer $150 for A, B, and C. Bids 3 and 4 become the provisional winning bids because the last time stamp among them is 2:30 pm, whereas for bids 1 and 2, it is 3:00 pm. If bidders can bid at multiple bid levels and are informed of prior bids, this rule will force new collections of non-conflicting bids to exceed old ones. If this is not the case, the rule is still desirable since it generalizes the current time stamp rule that encourages prompt bidding.

The rule generalizes in a natural "lexicographic" way as follows: use the second latest time stamp when the collections of non-conflicting bids are tied after considering the latest time stamp, and consider the third latest time stamp if the collections remain tied after considering the second latest time stamp, and so on. The use of the lexicographic generalization might be necessary, for example, if OR bids are allowed. If the last bids in two tied collections are alternatives in the same OR bid and the OR Bid has the latest time stamp, the rule would compare the second latest time stamps. If these both come from another OR bid, it would use the third latest time stamp; and so on. (However, we don't recommend allowing OR bids; see 12. below.)

8. Ties Should be Avoided

In spite of the fact that we have provided above a foolproof rule based on time stamps for breaking ties in this auction, economic ties are undesirable. In achieving economic efficiency, "first to bid" is a poor substitute for "is willing to bid more." There are a number of steps the Commission can take to make ties less likely or to mitigate their importance. Among these are the following:

The Commission can allow bidders to add an amount smaller than one bid increment to a bid. This amount would be a binding part of the bid, would not be reported, but would be used to break ties. The Commission could decide, in advance, either to use or not to use such amounts in setting the next bid increment. Use of this secret extra amount would be optional; it would encourage competition; and it would avoid the use of trailing digits for communication. We strongly recommend this alternative.[12]

Another thing the Commission can do, whether or not it accepts the above suggestion, is to allow more rather than fewer multiple increments. This should be done for individual licenses. In order to limit the "threshold problem," it should not be done for the global license. An intermediate number of increments should be allowed for regional and national licenses. One possible rule of thumb for this auction is to make the number of eligibility-conferring increments available for a license inversely proportional to the number of individual licenses it contains. Using this rule, the global license could be allowed only one increment, the national licenses two, the regional licenses six, and the individual licenses twelve. In addition, retained bids that are not provisionally winning may have a limited number of increments that are smaller than eligibility-conferring ones. We recommend this as well.

Another thing the Commission can and should do is to take the presence of ties into account near the end of the auction in setting the minimum bid increment. If there

[12] We realize that in early bidding rounds it is possible that many bidders will add the largest allowable sub-increment amount to their bids, thus causing ties. However, at the close of the auction when ties are important, this is unlikely.

are ties, the new required increment should be smaller. This will encourage competition to replace speed.[13]

One thing the Commission should not do to avoid ties is to use actual regional populations rather than rounded populations in defining a licenses' size for the eligibility calculation purpose. This could lead to rather arbitrary "one way streets," causing the bidding on some licenses to close before that on others for reasons that have nothing to do with their importance. This is undesirable.

9. Initial Bids

The initial bid on a package can and should equal the initial bid on its components except that the Commission may wish to add small amounts to the minimum bid for individual licenses and still smaller amounts to national and regional packages.[14] These additions would help to avoid ties and to counteract the threshold problem without creating arbitrary one way street problems.

The Commission can and should set initial bids below a reasonable estimate of the value of a license. Having done so, the Commission should neither lower nor raise the initial bids on licenses that do not attract bids. The failure of a license to attract bids under these circumstances is a warning to the Commission. If it turns out that a license does not sell in a particular sale, the Commission should plan on reevaluating both the license and the method of sale before re-offering it.

10. Auction Entry Rule

While the Commission did not call for comments explicitly on it and it is not restricted to combinatorial auctions, the present rule for auction entry is in need of improvement. The Commission's auction rules use a multi-round format that is designed to let bidders react to what is happening. However, when it comes to auction entry, the Commission rules allow no such reaction even though experience shows that entry, as reflected in eligibility ratios, has a profound effect on auction results. Bidders' initial eligibility is determined by their deposits, and no bidder knows whether other bidders will make deposits or how much they will deposit. Thus, bidders cannot react and adjust their entry decisions. Even thought the prices in the FCC auctions are heavily influenced by the eligibility ratio, these ratios are not known to the bidders when they make their deposit decisions. In particular, it appears that the low prices in D, E, F block auctions were a result, at least in part, of potential bidders deciding not to participate after observing the prices in the C block auction. Had the potential bidders known the low level of competition they would face, some of them may very well have been willing to participate.

To provide bidders with the ability to adjust to competitive levels, we suggest that the Commission change its rules to establish a late entry date. At this late entry date, entrants and non-entrants should be allowed to increase their eligibility, but at a

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[13] The trade off between efficiency and revenue on the one hand and, on the other, increment size and auction speed is explored in Michael H. Rothkopf and Ronald M. Harstad, "On the Role of Discrete Bid Levels in Oral Auctions," *European Journal of Operational Research* **74**, pp. 572-581, 1994

[14] If the Commission chooses to vary the initial bids on particular licenses or packages, it should be aware of the initial ranking of all 68 possibly winning collections of licenses/packages that this implies. It specifies a tie breaking rule that will persist whenever all licenses/packages are increased by the same percentage.

substantial penalty. The penalty, for example, might be to require double the deposit per unit of eligibility of late entrants or to subject late entrants to a 5% bidding premium.

If the Commission accepts our proposal made in 7. above to increase the deposit required of bidders, this proposal for allowing late entry will be even more important.

11. The Auction Pace Should be Managed Actively in a Way that Allows Bidders to Consult their Management

There are several aspects of the auction that can be actively controlled by FCC with the goal of ensuring the satisfactory pace and an efficient outcome. The commission has some experience with previous auctions, but the new format requires careful monitoring of the bidding process and active immediate adjustments of auction parameters, if necessary. In particular, among the parameters that can be adjusted on the round-to-round basis are the number of rounds in a day and the minimum bid increment. As for the stopping rule, we like the proposal to end the auction after two consecutive rounds without new activity credit granting bids. Note that the stopping rule can also be controlled indirectly, by controlling the minimum bid increment (which defines activity credit).

While the Commission did not call for comments explicitly on it and it is not restricted to combinatorial auctions, a new explicit rule for pacing the auction in the face of large bid increases is appropriate. The Commission should select a large amount, say $1 billion, and announce in advance that no new auction rounds will begin on any day in which the total of the provisionally winning bids has increase by that amount or more. This will help the bidders deal with authority delegation problems, and will not, if the selected amount is large, significantly slow the auction down.

Conversely, in a combinatorial auction, when the only bid increments received in a round are increases in retained but non-provisionally winning bids, the next round should follow with minimal delay. The only exception should be if no new bids at all have been received in a round, and hence, there is risk that the auction will end on the next round.

We favor the Commission retaining the *right* to accelerate the auction by announcing a limit on the number of future rounds, but we counsel extreme reluctance and caution in the exercise of that right.

12. Don't Use OR bids

We do not favor allowing the use of OR bids. They should certainly not be introduced in the first FCC auction that will allow combinatorial bidding. As we have noted above, insisting a bidder have eligibility only for the maximum possible winning combination of its bids rather insisting on eligibility for each bid greatly reduces the incentive for allowing OR bids. Furthermore, allowing for withdrawals of retained bids that are not provisional winners, as described in 5 above, is a much more natural way to deal with the exposure problem. Also, because of the multi-round format, bidders who are not constrained by a rigid budget and who are not behaving strategically, should have little need for OR bids.

Special caution on OR bids at this time is called for because the Commission's Request for Comments shows that it does not yet fully understand the issues with respect to them. First of all, the Commission asks if it should forbid or allow OR bids involving

a regional and a national package. There is no need to for bidders to use OR bids involving a regional and a national package since such bids are necessarily in conflict with each other and therefore cannot both win. The Commission has neither a need to forbid the use of such OR bids nor a reason not to. Secondly, the Commission asks about allowing more than two alternatives in OR bids. If, against our advice, the Commission decides to allow multiple OR bids, it makes little sense to disallow OR bids involving multiple alternatives. If a bidder makes OR bids on the pairs A,B, B,C, and A,C, she has effectively made an OR bid on the trio A,B,C. It would probably be simpler for all involved in such a situation if the bidder just specified A OR B OR C.

Furthermore, proposed rules on which part of OR bid to retain could easily lead to inconsistencies and suboptimal outcomes. In fact, any proposal that does not retain all parts of an OR bid that would be retained if they were submitted as a regular bids faces a risk[15]. It incurs a chance that at some later stage of the auction exactly the non-retained part could have been a provisional winning bid. There is a (highly non-transparent) way to deal with the issue of retaining OR bids in an inconsistency-free and computationally manageable way. We could elaborate on this procedure, if necessary.

If the Commission, against our advice, decides to allow OR bids, we would like an opportunity to suggest means for doing so.

Here we mention one form of OR bids that might be more palatable for small bidders: Some bidders, particularly smaller ones whom the Commission has been directed to favor, may have a rigid budget constraint. Whatever is done about OR bids in this auction, the Commission should investigate the advisability of allowing bidders, or perhaps just small bidders, to submit budget constrained bids in future auctions.[16] Such a constraint would be taken account of in the calculation of the revenue maximizing set of bids.

For example, suppose a bidder submitted bids of $10, $20, $25, $30, and $35 with a budget constraint of $60. She could not win any combination of bids that totaled more than $60. Thus, any one bid, any pair of bids except the two largest, the $10, $20 and $25 bids, or the $10, $20 and $30 bids could win, but no other combination of her bids could. In any round, the revenue maximization calculation would accept as provisionally winning whatever allowable combination of her bids leads to the maximum total sale revenue. Ties would be broken by time stamp. If such budget-constrained bids are allowed in future auctions, the activity rule would have to be adjusted to account appropriately for them. In addition, serious thought will need to be given to a rule on whether bids subject to a budget constraint that are not provisionally winning bids are retained for subsequent rounds.

In addition, the Commission should realize that adding budget-constrained bids has the potential to make winner determination in large simultaneous auctions computationally unmanageable. (This may not occur in practice, especially in auctions with few licenses, and need not deter the Commission if it has an adequate plan for

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[15] Both the proposal in IIIA and the proposal in footnote 42 are of this type

[16] Budget limits do matter. See for example Yeon-Koo Che and Ian L. Gale, "Standard Auctions with Financially Constrained Bidders" *Review of Economic Studies*, **65** (1998), 1-21. We know something about how budget constrained bidders bid in the absence of synergies when they are not allowed to bid with a budget constraint. See Michael H. Rothkopf, "Bidding in Simultaneous Auctions with a Constraint on Exposure," *Operations Research* **25**, pp. 620-629, 1977. More research is needed on bidding with a budget constraint in the face of synergies.

dealing with this contingency if it does occur. For example, if the Commission finds it cannot handle the budget constraints computationally in a given situation, it might require those who used them to rebid that round without them.)

13. Do Not Restrict Package Bidders from Later Bids on Package Components

The bid increment calculations and the activity rule we propose should deal adequately with the threshold problem. Thus, restricting bidders who are high bidders on a package from bidding subsequently for a subset of that package is not necessary and could be counterproductive. It limits bidders' flexibility. For example, it is possible for a bidder to have a strong synergy for a regional package as well as a high valuation for the 10MHz license in that package. It is possible that during the course of the auction an unexpectedly strong 20MHz national bid occurs and that that bid, together with the six individual 10MHz license bids, form the collection of provisional winning bids. It is also possible that the threshold problem faced by the bidder interested in the regional license is insurmountable, and thus the bidder would like to at least get the 10MHz license in the region. However, the proposed bid composition restriction rule will not allow for that, thus possibly yielding a suboptimal outcome.

Another problem with the proposed bid composition restriction rule is that even if the bidder's intentions were as described in IIIC, the proposed rule might still not stop the bidder from doing exactly what the rule is designed to prevent. Suppose that a bidder bids on a national 20MHz license and decides she wants to bid only on some individual 20MHz licenses. The proposed rule prevents the bidder from submitting a bid on any individual 20 MHz license. However, the bidder is free to submit bids on any regional package. It is possible that some of the 10MHz individual licenses will be cheap compared to the difference in the amount the bidder is ready to pay for the 20MHz licenses in the same regions and the current high bids on these licenses (especially if the price on 20MHz license is low, and this could well happen because, by submitting an early provisional winning national 20MHz bid, the bidder successfully deterred bidding on individual 20MHz licenses as well as the bidding on regional packages). Then, even with the proposed rule in effect, the bidder could simply submit regional package bids covering 20MHz licenses she is interested in. Furthermore, the proposed rule might further help this bidder by deterring some other bidders from submitting competitive bids for regional packages because they want to have an option to bid on individual licenses later. Thus, the proposed rule could work directly in favor of the scenario it is designed to prevent.

14. The Commission Needs Appropriate Advice

The Commission should recognize the emergence of new breed of issues that combinational bidding brings. In particular, while standard strategic auction theory considerations remain crucial, the issues dealing with underlying combinatorics and combinatorial optimization are also important. These two aspects are fundamentally interconnected. Neglecting either of the two (e.g., by seeking advice on combinatorial issues from auction theorists or by seeking advice on auction theory issues from experts in combinatorial optimization) is a formula for potential disaster.

15. In the Future, Acknowledge Seminal Work on Combinatorial Auctions

The Commission can rest assured that the proposed allowable combinations of bids will not cause undue computational difficulties when the Commission seeks to calculate the revenue-maximizing set of provisionally winning bids. This is assured by the analysis in Rothkopf, Pekec, and Harstad.[17] This paper was written directly in response to the Commission's earlier concern that combinatorial auctions would be computationally unmanageable and shared with it, in working paper form, as early as 1995.[18] It would be appropriate in future notices for the Commission to acknowledge explicitly relevant work such as this done *pro bono* and not just the work of its paid contractors.

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Durham, NC and Piscataway, NJ
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[17] *Op. Cit.* Example following Theorem 9, p.1141 proposes (and discusses the implementation of the procedure for determining winners) in the very combinational auction format that is now being proposed by FCC. Also note that the FCC's proposed rule for retaining bids is governed by the principle presented in Observation 3, p. 1137.

[18] Michael H. Rothkopf, Aleksandar Pekec and Ronald M. Harstad, "Computationally Manageable Combinatorial Auctions," RUTCOR Research Report #13-95 and DIMACS Technical Report 95-09, Rutgers University, New Brunswick, N.J., 1995.

Appendix: Criterion for Eligibility-Conferring Bids

Here is a mathematical statement of the criteria we are suggesting for the minimum bid increment on a biddable combination (where combination includes the combination of a single item).

Let c be a biddable combination.
Let b(d) be the current bid on combination d for any biddable d.
Let W be the set of provisionally winning combinations.
Let R(S) be the revenue from accepting bids on a set of combinations S, i.e.,

$$R(S) = \Sigma_{d \varepsilon S}\, b(d).$$

Let Z_c be a set of combinations that contains c and is non-conflicting.
Let $L(Z_c) = \Sigma b(d)$ where the summation is over d belonging to Z_c but not to W.
Let

$$B(c) = \text{Min over } Z_c \text{ of } [R(W) - R(Z_c)][b(c)/L(Z_c)]$$

The minimum eligibility-conferring bid increment on c should be max{B(c),X}, where X is the increment decided by the FCC for the next round. Note that when c is a provisionally winning combination, B(c) is 0.[19]

[19] We also note that B(c) is a first approximation lower bound on the minimum bid increment $B_{min}(c)$ ensuring that there exists Z_c maximizing R(S) over all possible nonconflicting sets of combinations S, provided that b(d) is increased by $B_{min}(d)$ for every biddable combination d. The method for computing $B_{min}(d)$ is more complicated, but computationally manageable as long as the problem of determining auction winners is computationally manageable. We can elaborate on this method upon request.